8. Münchner Wissenschaftstage

18.-21. Oktober 2008

Arithmetics and Weaving
From Penelope‘s Loom to Computing

The Loom of Ancient Greece

Weaving means to stretch out threads parallel (the warp) and to pass another thread (the weft) by going over and under each warp-thread. Often a rod (shed bar) is used to lift every other warp-thread which makes the work a little bit easier.

Today warp-threads of great length are wound around a warp beam and the cloth is produced ten marks a meter and rolled up continually. From early depictions of ancient looms we may learn that warp-threads were not rolled up this way. Furthermore they are fixed on the same beam where the cloth is hanging from. Some amounts of warp-threads are gathered and stretched out by a weight that gives the tension that is necessary for weaving. The loom is standing upright and was named *histos orthios* (*tela recta* with the Romans*)* literally speaking: (up)right loom.

The following figure shows a diagram with the most important parts of this loom and their names. If the heddle bar (*kanon*) is not lifted and rests on the warp there will be a natural shed as one half of the threads will be held by the lower rod named *kairos*. If the weaver lifts the *kanon* all the other threads will pass through and make the countershed.

The special textile meaning of the word *kanon* (heddle bar) as part of the loom ist much older than the transferred sense (canon, rule) and can be found as early as Homer‘s *Ilias*. The *kanon* provides the regular structure of the weave and his name/word was transferred to other tools for ordering and regulating as they are used by musicians and architects. The latin word for heddle bar: *regula*, is known as basis of the words „rule“ and „ruler“ down to the present day. And the shuttle, *penion* in Greek, is *radius* in Latin. To fix the warp-threads to the *kanon* commonly a stable thread of linen was used, named *linum* which we nowadays find in *line* (Latin: *linea)* and the German word „Lineal“ (Latin: *regula*).

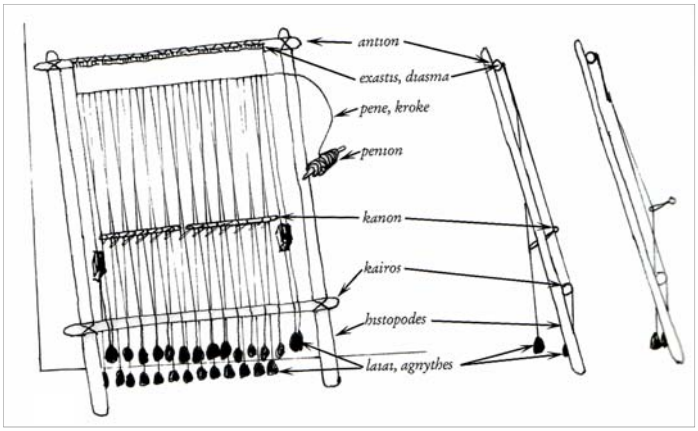

Diagram of a Greek warp-weighted loom with names of important parts.

The Relevance of Ordering

Doing the warp is called *ordior* in Latin and is the basis of all words used to speak of ordering or sometimes even laws (ordinance). The French word for computer: *ordinateur*, is derived from this word for warping threads.

If one knows about the complexity of weaving patterns and pictures it is no wonder that weaving terminology and recent terminology on ordering, regulating, and measuring are connected. At first sight the loom of antiquity looks primitive, but it works not only in making simple weaves but even patterned ones, pictures and double weave. One of the oldest finds of warp-weighted-loom depictions up to date shows a very complicated pattern with geometric motives. But in weaving every geometric motive assigns an arithmetic task: it has to be transformed into number ratios according to the dualistic up and down of the warp-threads. Furthermore it has to fit into the width of the warp if it is to repeat in an aesthetic way. Therefore pattern weaving requires good knowledge of rules for divisibility of numbers. And if the density of warp and weft ist different you have to master the rules of proportionality too.

Cyprian plate with depiction of a loom, ca. 850-750 b. C, Antikensammlungen University of Bonn

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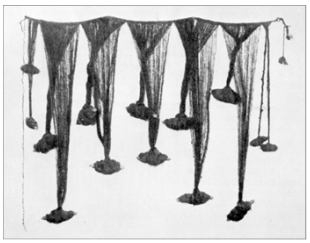
Distinctive Features of the Heading-Band

A distinctive feature of the warp-weighted loom makes patterning the cloth at once difficult and easier. The warp-threads are not fastened directly to the warp-beam. First a band was woven as long as the later cloth should be wide. The weft of this band is stretched out on one side of the band as long as the later cloth should be long. This band with the loose threads was attached to the warp- or cloth beam and the former weft was used as warp by stretching the threads by loom weights.

By weaving the band the warp-threads have been grouped: the even ones and the odd ones in separate bundles as you may learn from a textile find of Stavanger in Norway (see below, left). The band was integrated into the weaving and elongated as selvedge. When the cloth was ready there was no need for cutting. Hems and edges are an integral part of the fabric from the start of weaving.

That means the patterns have to fit into this heading-band from the start. And it means that the weaver has to consider what follows for patterning the whole cloth. If the number of weft threads in the heading band (and this is the number of warp threads for the cloth) ist a prime number, no repeat of pattern will ever fit. And it is evident that warp-thread numbers with much divisors will do best.

Even if the method of using a heading-band to us looks like a handicap on first sight it might be thus helpful. Most textiles of antiquity show elaborately decorated borders. In some cases you may read from such borders directly the divisor of the number of warp-threads, because it is a multiple of the threadcount of the pattern (if the pattern is woven completely and does not break in the end). A reconstruction of such a fabric was done in the exhibition „Penelope rekonstruiert“ and the band has a running dog with a pattern of 8 threads. Are they woven without break the weaver knows that the later warp-threads will be divisible by eight, four and two and he may choose an according pattern for the cloth.



Heading-band from Stavanger

Reconstructed heading-band with running dog

Dyadic Arithmetic

In Greek Antiquity there was a special kind of arithmetic that makes it very easy to handle such problems of divisibility: the arithmetic of odd and even numbers, sometimes called dyadic (that means: two-value) or pythagorean arithmetic. Therein numbers are classified by their characteristics in divisibility and the theorems of this theory tell us how to generate numbers with certain divisibility-characteristics or they tell us how to make conclusions on the characteristics of the generating numbers from an existing one.

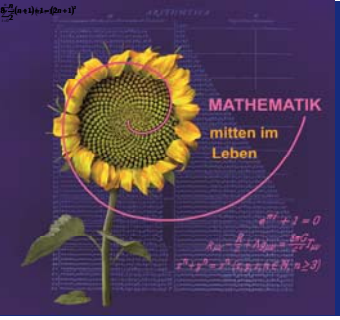
It is assumed that the philosopher Pythagoras developed this number theory out of harmonies in music and Iamblichos tells the following story on this discovery: Pythagoras once walked by a forge „where he heard by a divine chance hammers beating iron on an anvil, and making mixed sounds in full harmony with one another, except for one combination.“

Becoming curious on the cause he went in, weighed the hammers, went home, and started the following experiment: „From a single peg fixed to an angle between two walls … he suspended four strings of the same material, of the same number of strands, of equal thickness, and of equal torsion. And from each string he hung one weight by attaching the weight at the bottom and making certain that all the strings had equal length.“

This description will also fit as a description to build up a warp-weighted loom. And the italian musician Gaffurio imagined this instrument of discovering harmonies in just this way (see picture above). It looks like a warp-weighted loom lying on a table for better handling.

But the nice story told by Iamblichos is not able to explain the discovery of all the theorems and proofs of dyadic arithmetic that are handed down to us by Euclids book *Elements*. Number representation in times of Pythagoras made no use of a place value system that makes it easier to discover divisibility rules. Even here a cloth on a loom with its countable pattern-threads would be more convenient than the letters the Greek used for numbers. Was the warp-weighted loom the starting-point of dyadic arithmetic?

Pythagoras assumed that all things wer made of numbers. This concept of numbers has already been a questionable issue in antiquity. Aristotle made fun of this when he wrote on the Pythagorean called Eurytos that he „determined what is the number of what object and imitated the shapes of living things by pebbles after the manner of those who bring numbers into the forms of triangle or square“. These pebbles or *psephoi* as the Greek called them, are held to be the provenience of dyadic arithmetic.



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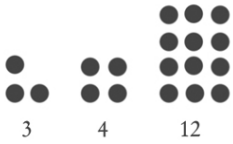
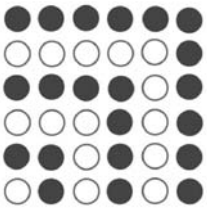
The Pebbles



How have these pebbles been used? According to Thales a number is a texture of units. So the pebbles may be used as units to build a texture of a certain number of these units and to give a certain shape or form to this texture.

Then you may name numbers after the forms which you can build of them. For example triangle numbers (3, 6, 10 etc.), square numbers (4, 9, 16 etc.), oblong numbers 6, 12 etc.) and so on.

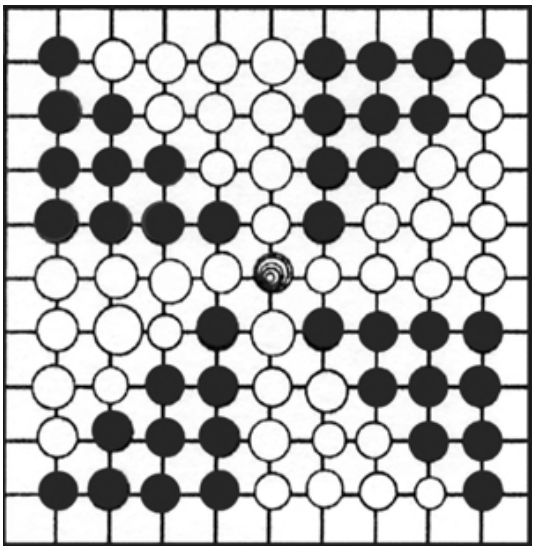
According to the arrangement of the pebbles or their grouping by shades (dark or light) it is possible to demonstrate quite complex issues that are described nowadays by algebra. In Antiquity there was no chance to do this in writing due to the lack of ciphers and place value system.



The summation formula $\sum_{i=0}^{n} 2i+1=(n+1)^2$ for example may be represented as a pebble texture that makes it (more) easy to see the formula as a combination of odd numbers starting with the unit: The sum of successive odd numbers beginning with one is a square number. The Greeks had to say (see) it with this words because they had no algebra in the modern sense.

With Philolaos we find another example that may be represented by a pebble texture: Each eightfold triangle number is by one unit smaller than a square number. Today we would write:

$$8\cdot\frac{n}{2}(n+1)+1=(2n+1)^2$$



Some Definitions and Theorems of Dyadic Arithmetic

Some definitions

Def. 1 Unit is (that) according to which each existing is named one.
Def. 2 And number (is) a multitude composed of units.
Def. 6 Even is the number (which can be) divided in half.
Def. 7 And odd is the number (which can)not be divided in half, or which differs from an even number by a unit.
Def. 8 Even-times-even is the number (which is) measured by an even number according to an even number.
Def. 9 Even-times-odd is the number (which is) measured by an even number according to an odd number.
Def. 10 Odd-times-odd is a number (which is) measured by an odd number according to an odd number.

Some theorems (propositions)

Prop. 21 If any multitude whatsoever of even numbers is added together then the whole is even.
Prop. 22 If any multitude whatsoever of odd numbers is added together, and the multitude of them is even, then the whole will be even .
Prop. 24 If an even (number) is subtracted from an(other) even number then r the remainder will be even.
Prop. 31 If an odd number is prime to some number then it will also be prime to its double.
Prop. 32 Each of the numbers (which is continually) doubled, (starting) from a dyad, is an even-times-even (number) only.
Prop. 33 If a number has an odd half then it is an even-times-odd (number) only.
Prop. 34 If an number is neither (one) of the (numbers) doubled from the dyad, nor has an odd half, then it is (both) an even-times-even and an even-times-odd (number).

Aias and Achill doing a board game – perhaps with pebbles? (Drawing from an antique vase)

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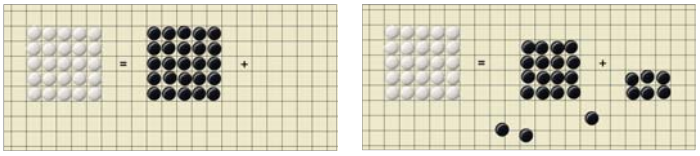
Incommensurable Quantities and the Indirect Proof

The greatest achievement of Greek mathematics is to give it a straightforward logical and formalistic system. Crucial to this are two discoveries or inventions: the method of indirect proof and the discovery of incommensurable quantities: in nowadays terms the fact that there are ratios that may not be representable by integers (what we now call irrational numbers). The best known example is the ratio of side and diagonal of a (unit) square. This incommensurability is proved within Euclids Elements. And this proof works indirect by using the dualistic number properties of even and odd.
Assume that there is a common measure of the square-side a and the diagonal b. Then you can deduce by the theorem of Pythagoras (in this case: $a^2 = 2b^2$) that b is as well even as odd. And this contradiction proves that the opposite of the assumption is correct.

The proof makes use of the presupposition that two numbers might be reduced to terms that are prime to each other. This is not an easy condition and mathematicians assume that there has been a pebble proof for the incommensurability in the square that worked without it.
In this case you will have to double a square as to make a new square or otherwise divide a square into two squares of equal size.



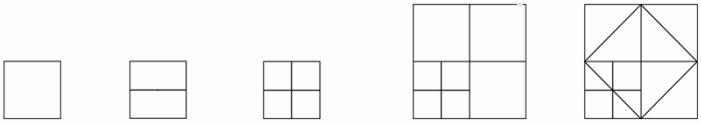
Let‘s have a Geometry Lesson with Socrates: Double a Square!

The famous „Geometry-Lesson“ in Plato‘s Dialogue *Menon* gives just this type of Square-doubling. An untaught boy or slave has to solve this task and Socrates wants to show by this example that mathematical ideas are innate in everyman.

Socrates draws a square of two feet side-length (see first drawing on the right), divides it into two halfs (drawings 2 and 3), and asks for the area. The boy answeres: 4 feet (we would say 4 square-feet). Now he is asked to double the square and he answeres correctly: 8 feet, but for the drawing he proposes the doubling of the side-length. Socrates now gives the hint that the square-area will then be 16 feet (Zeichnung 4) and the boy reduces the length to three feet. But for the area this gives 9 feet and in the end the boy assigns: „It won‘t work.“

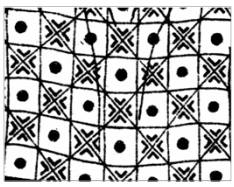
The solution is only found by tha diagonal that Socrates draws now. This makes 4 triangles of two feet in area. These are the eight feet searched for and the diagonal is the according length.

This lesson not only shows how the problem is solved geometrically. It also shows the difficulty to detect a number for the length of the diagonal. But Socrates does not mention this Problem.

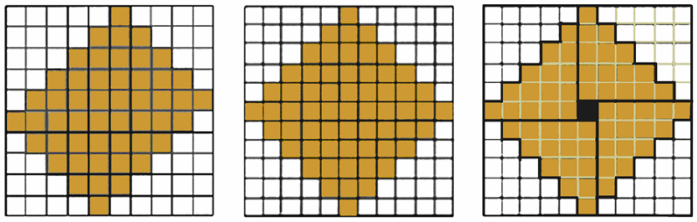


To daoble a Square in Weaving

Let us suppose the boy went home with pride in his new knowledge and meets his sister with her selfmade dress showing just the figure of the lesson he survived. Now he wants to know the answer as number, counts the thread crossings – and the numbers are wrong.
His sister knows the reason: If she plans the patter she has to start with the four-times Square of Socrates‘ drawing, giving the width of the motiv.



The thread count of this repeat is even because it is a double length. But with an even number of threads she may not weave the spearheaded square in the center. There is no thread in the middle to make a thread cross as starting point of the diagonals. (see below, left drawing).
But if the thread count of the repeat is odd (what will be contradictory to the double length), you can make a perfect solution (see below, drawing in the middle). But the spearheaded square has one thread crossing too much – and this was what the boy hat counted.



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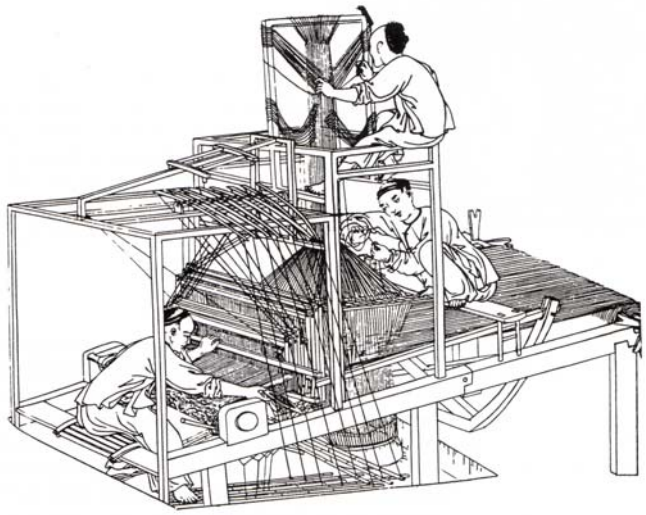
Leibniz and China

In january 1697 the mathematician and philosopher Gottfried Wilhelm Leibniz sent a letter to the emperor of China describing a dyadic arithmetic of zero (nihilum) and one (God‘s word). He wrote: „Anything may be solved with this method.“ And he also prefigures „that with this dyadic one can build a computing machine.“

All weaving is done in dyadic terms. When the weft comes across the warp there is only to chose between zero (warp-thread down) and one (warp-thread up). And as damask weaving shows: anythig may be depicted with this method.

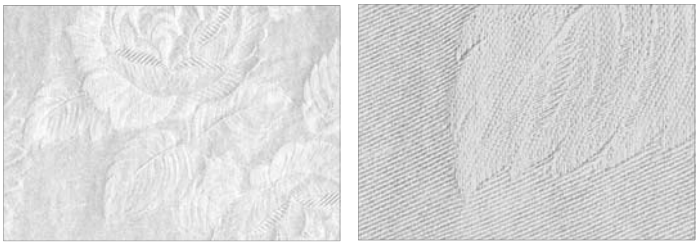
As early as the first centuries AD pictured cloth was produced mechanically in China on drawlooms. They work with an additional thread system that allows to lift certain groups of warp-threads. A draw boy was necessary, sitting on the harness and lifting the according thread after each treadle the weaver used (the third person in the picture below has to repair broken warp threads).

Italian workshops for weaving silk have been famous for this Drawloom-Work in the 15. Century. Not only the drawboy makes it costly. At first there has to be drawn a draft or point paper design with every single thread cross. Then the loom is set up according to this point paper design, that means the warp is set up with the shafts and the harness. The threads of the harness are grouped according to every weft line. With complicated patterns the setting up of the loom sometimes took a whole year.



Calculated Flowers and Leaves

A cloth woven on such a loom may show flowers and leaves even though the whole textile is made of the same white threads. It is just the up and down of the warp-threadds that makes the picture as effect. The rays of light break different when the direction of thread is different. And the flowers and leaves that we see are no accident but determinated by the countable and digital order of threads that is given by a the dualistic up-and-down principle.



Flowers and Leaves on a Damask Tablecloth. Detail on the right.

Jacquard and Babbage

Dinner parties at Charles Babbage have been an important meeting point for intellektuals in 19th Century London. Everyone knew he was working on computational machines and loves to present his advances on this occasions. On one of these soirées Babbage showed a portrait of the French inventor Joseph-Marie Jacquard and said: „This picture is very helpful to explain the nature of my calculating engine.“

Jacquard-woven Portrait of Joseph-Marie Jacquard

The picture was woven on a loom with a control system invented by Jacquard himself. This machine replaced the harness by a control instrument that used punched cards and even made the drawboy dispensable. Now one weaver alone could make these highly complex weaves without taking care of the pattern. All the complexity formerly hidden in the harness cords ist now controlled by the punched holes of cards.

Ada Lovelace who made econcepts for programming the second one of Babbage‘s calculating machines, the Analytical Engine, gave the following description of its function: it „weaves algorithmic patterns, just as the Jacquard-Loom weaves flowers and leaves.“

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Bröselmachine and Jacquardmachine

The Jacquardmachine war the most successful machine of this kind – but it has not been the first one. In the Mühlviertel in Upper Austria weavers have already worked without drawboy on the beginning of the 18th century. They used an wooden engine named Bröselmachine. Wooden logs (Brösel?) were glued onto a small tape of linen and this tape was scanned by metal needles to lift the according warp thread bundles.

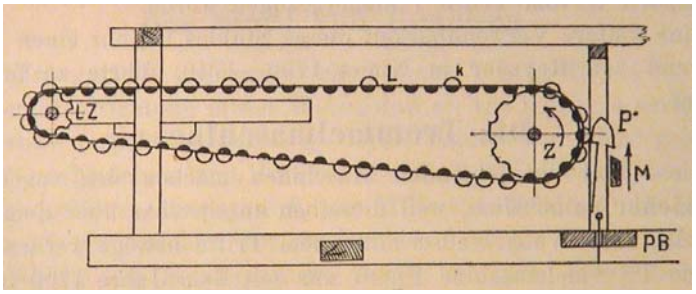
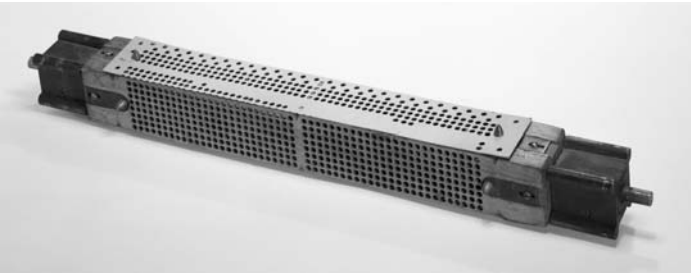


Diagram of Bröselmaschine after Kinzer

The Bröselmachine, Weaving Museum Haslach, Photo: Harlizius-Klück

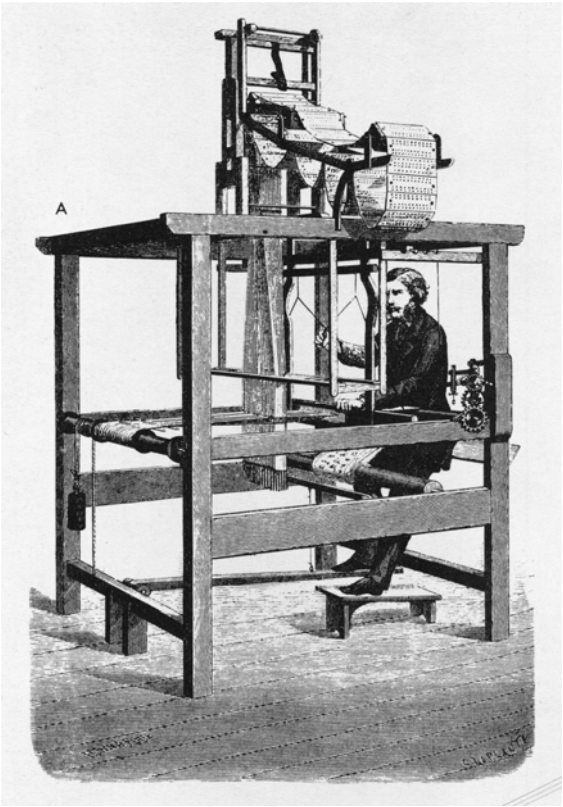


Jacquards punched cards are made with the same principle, but instead of the log the scanning of the metal platines reacts on the holes in the card.

Machines for Punching Cards

Punching the cards according to the point paper of a decorated cloth remains a time-consuming task. But the cards can be used repeatedly. It was the principle of punched tape that made Jacquards invention a success and proved to be highly effective for controlling machines. Herman Hollerith used the controlling by punched cards for statistical purposes on a large scale. And here begins the success story of modern computers that may represent almost anything on the basis of a dual code of zero and one. And similarly as woven pictures can do it since some thousand years by the dual code of lifting or leaving the warp-threads.

Machine for punching Jacquard-cards, Munichs Silk-Art Weaving Manufacture Gerdeisen, about 1920, Foto: Deutsches Museum, Munich



Loom with Jacquardmachine, The punched card are sewn together to a long tape rolling over the prism for scanning.